# Math Happenings!

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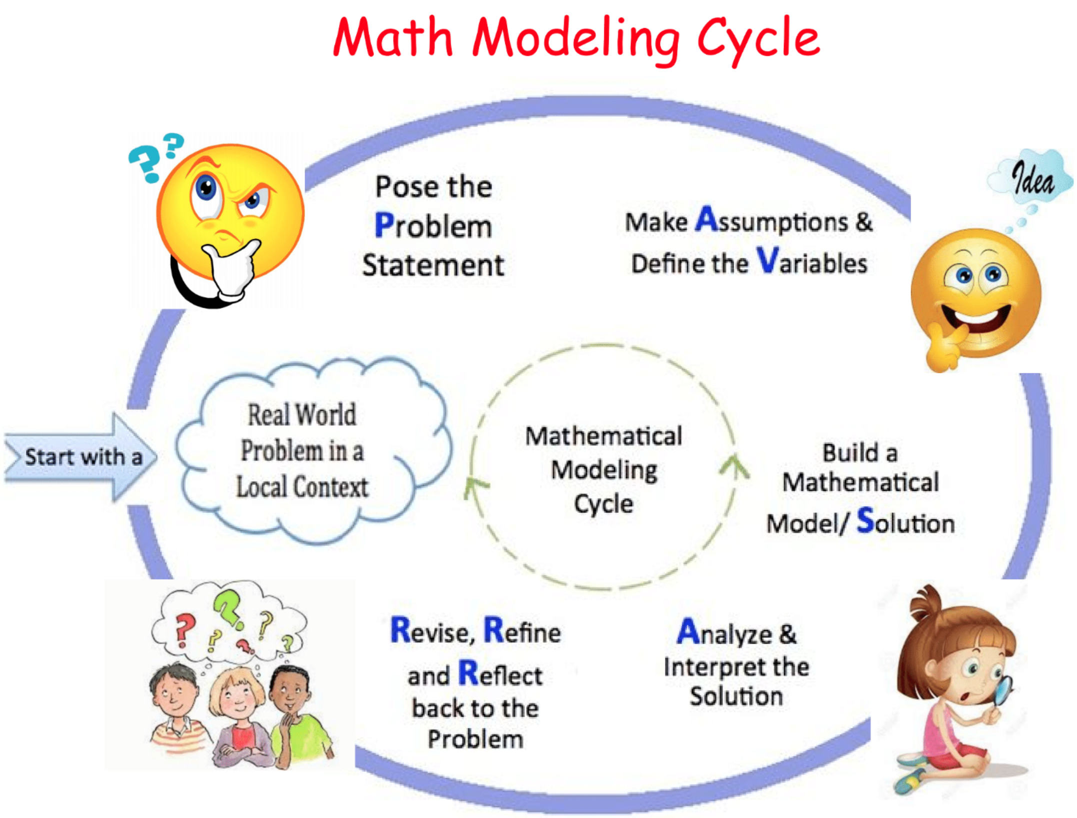
http://mathhappenings.onmason.com/

A math routine to introduce students to **mathematical modeling** as they build a relationship with mathematics. Students to see themselves as math doers and thinkers. They see the utility of mathematics and the relevance to their lives. Empowers our students as they see that mathematics can serve them.

**BACKGROUND INFORMATION:** Math happenings occur daily in all of our lives. The math happening lessons serve as a framework for teaching many mathematical concepts within the context of real-life math events. The teacher's role in the math happening lesson is:

- to encourage students to share stories about events that actually happen to them
- to interpret, translate, and represent these stories mathematically, using multiple representations
- to introduce other math concepts for which students are ready.




**OBJECTIVE: Model with Math**

Share a real-life event (math happening) and pose a question that can be answered using the information given in a math event from everyday life.

Key Processes- Problem Posing, Making Assumptions, Solving a Problem, Look for Patterns and Generalizations and Reflect the solution back to the Real world phenomenon.

**MATERIALS:** real world math materials, artifacts, photos, letter or an email. Read *The Math Curse* by Jon Sciezca and Lane Smith. This is a great read-aloud for students to experience all the math they experience in a given day at school.

**Getting Started:** Teachers can start with a story that happened to them in their life.

- Math happened to me. Let me tell you about it. (Tell the story. Talk outloud what you are trying to find out. What information do I need? Ask the question to help simplify the real world problem.)
- What math happened to you? Tell us about it. Tell me what you did last night, yesterday, or this weekend. (Listen to the event. Probe to gain enough information to make a math story and ask a question.)

Use this organizer to unpack the math! As more "math happenings" are shared in class, students will be able to better connect the math they are learning to their everyday encounters. Soon students will be coming to school after a weekend and saying, " I had a math happening this weekend…."

# Math Modeling Planning Guide

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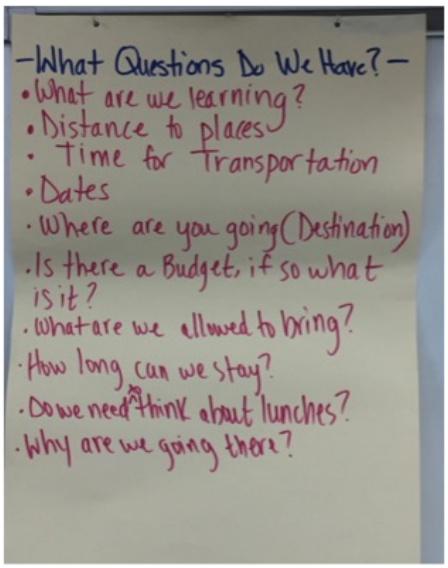
| **Problem Posing**<br>*Sample Prompt:*<br>• *What do you wonder about?*<br>• *From the list of wonders, how can we use math to make a decision? How can math serve our needs?*<br>• *What do you notice? What do you wonder?*<br>• *What is interesting or important about this situation?*<br>• *Who cares about this situation?*<br>• *What other information do you need? Are there important quantities?* | Students were asked to recall their past experiences with field trip to make personal connections.<br><br>Students were given an article- *How Field Trips Boost Students' Lifelong Success- with tips for planning a successful trip.*<br><br>Set the stage for MM by sharing the phenomenon-<br>"The principal has asked us to plan a class fieldtrip. How will we plan for this?"<br><br>Allow students time to brainstorm. Students asked about:<br>Budget<br>How many people?<br>How will we get there?<br>How long will be there?<br>Who comes with us?<br>Does it have to be academic?<br><br>Monitor student progress and group dynamics.<br>Listen for:<br>• ideas that help students mathematize the problem<br>• common misconceptions<br>• Note which should be shared with the class. |
| **Making Assumptions**<br><br>*Sample Prompt:*<br>*Gathering Information*<br>• *If I knew______, then I could figure out_______*<br>• *Let's assume__________.*<br>• *What assumptions can we make around the problem to simplify the problem posed?* | Costs of transportation- provided by teacher<br>Content covered throughout the year (year @ a glance)- provided by the teacher<br>Total # of individuals attending- provided by the teacher<br>Budget (what's reasonable)- provided by teacher<br><br>Needs to be academic- related to the content we learn<br>Local- needs to be done in a day<br>Budget- reasonably affordable |
| **Doing the Mathematics**<br>*Sample Prompt:*<br>• *What math do you know that can help you solve the problem?*<br>• *What are some common misconceptions that could arise at this stage, and how can they be addressed?* | *Food costs*<br>*Transportation costs*<br>*Relevancy*<br>*Unexpected contingency plan (Weather? Place to eat?)*<br><br>*Note misconceptions and select ones to address individually or as a group. During the lesson, occasionally regroup the whole class to share student ideas, provide additional information, or add/discuss constraints.* |
| **Building a model**<br>*Sample Prompt:*<br>• *How might we use our solution to other similar problems?*<br>• *Can this solution help us with other problems?* | *What components do you expect the students' models to include?*<br>Food costs<br>Transportation costs<br>Event costs<br>Connection to curriculum<br>People |

| | |
|---|---|
| | *What will a useful generalizable model be able to do?*<br>A useful model will be able to represent the total cost of their planned field trip, AND it can be applied to any future trip. |
| **Evaluating and revising the model**<br>*Sample Prompt:*<br>● *How might we become even more precise with our model?*<br>● *How might our model change if we changed some of our previous assumptions?* | If students were planning any additional trips (at home or at school), they will hopefully have the model in place to do so. The basics of transportation costs + event costs + food costs = total costs, which can then be divided out to a per person cost. This model can be applied in the future to planning models. |
| **Sharing & Connecting Real world Math to school math**<br>*Sample Prompt:*<br>● *How does all the math we used in this problem relate to what we covering as math topics this year?*<br>● *Can you identify all the math and strategies we used to tackle this problem?* | *In what format will students present?*<br>*Students created a google slide presentation that was shown to classmates, admin and fellow 5th grade teachers.*<br><br>*What expectations do you have for students' presentations?*<br>*Students will follow the rubric to make sure they've included:*<br>*Academic link*<br>*Math model*<br>*Budget expense report* |




## CONTEXTS IDEAL FOR ELEMENTARY GRADES

| | |
|---|---|
| ● Community-based/Service learning MM projects | Food for Thought- Helping homeless<br>Coin Harvest Fundraiser<br>Designing a Community Center<br>Bedtime Snack Pack |
| ● School-based MM projects | School supply distribution<br>Lunch count<br>Field trip Planning<br>Designing Ideal learning environment<br>Planning a classroom party<br>Class pet acquisition |
| ● STEM/STEAM contexts | WInd-powered Car<br>Lego Cars<br>Marble Run<br>Emergency Plan for Natural Disasters |
| ● Interdisciplinary contexts not STEM | Best Olympic Athlete MM Task<br>Electric City Construction/electrification |
| ● Social Studies/Current Issues | Mock Election; City Project, Grade level Olympics (on Olympics years) |

# Gathering Information

### If I knew______, I could figure out_______

(For example, If I knew the cost of a school bus for a field trip, then I could figure out the cost of the transportation.

If I knew how many students and chaperones were coming on the trip, then I could figure out the total number of school buses we would need. )

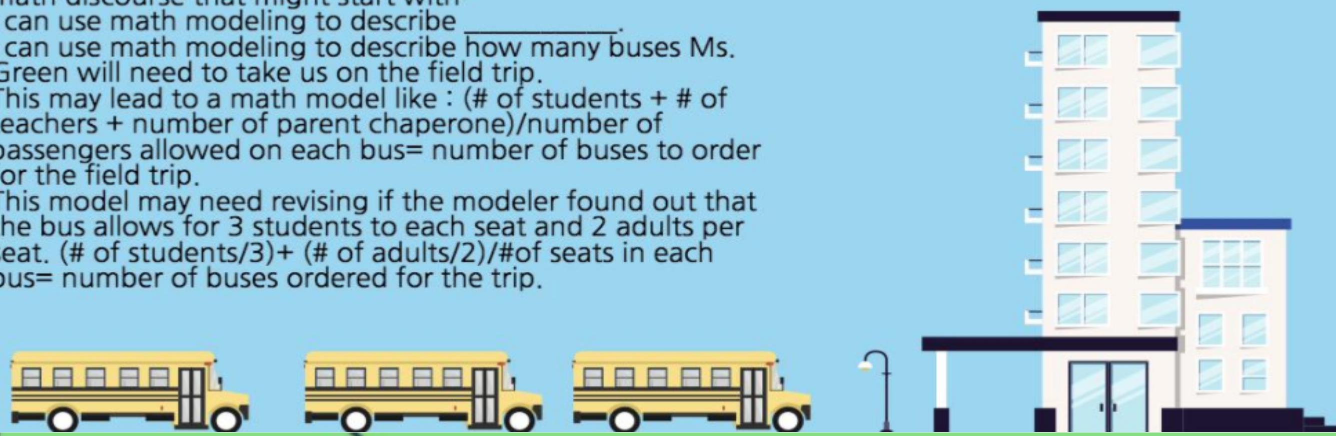
## Describing a math happening

I can use math modeling to describe ...

# Descriptive Model

Using real world data to describe/represent/ analyze a phenomenon

For emergent math modelers, this would engage them in math discourse that might start with-
I can use math modeling to describe ___________.
I can use math modeling to describe how many buses Ms. Green will need to take us on the field trip.
This may lead to a math model like : (# of students + # of teachers + number of parent chaperone)/number of passengers allowed on each bus= number of buses to order for the field trip.
This model may need revising if the modeler found out that the bus allows for 3 students to each seat and 2 adults per seat. (# of students/3)+ (# of adults/2)/#of seats in each bus= number of buses ordered for the trip.



## Making Predictions

I can use math modeling to predict ...


# Predictive Model

Using trends and data analysis to predict an outcome
For emergent math modelers, this would engage them in math discourse that might start with-
I can use math modeling to predict ______________.
I can use math modeling to predict how many pencils we will sell at our School Store based on our data.
With our Buy one get 2 free pencils sale, if we make at least 5 sales, that would mean
number of pencils sold=5 +2(5)=15. Some students might be able to say it in words or with a number sentence, while students ready for algebraic reasoning might be able to use variables to represent the model.
P=5+2(5)
P=n+2n

Finding the "best"

I can use math modeling to find the "best"...

## Optimizing Model

Using data to find the "best" by optimizing or in some cases minimizing some situation.

For emergent math modelers, this would engage them in math discourse that might start with-
I can use math modeling to find the best ______________.
I can use math modeling to find the "best" way to design an edible garden.
For optimizing models in earlier grade, it provides a great opportunity for students to engage in mathematics argumentation because the criteria for "best" can be determined by the assumptions and constraints they put around the real world problem. For example, Ms. Farmer wants to maximize area with a plot of land and has 24 feet of fencing or Ms. Farmer has a narrow plot of land that is only 4 feet wide but had lots of room for a long garden or Ms. Farmer wants to use the wall of her house as one side of the garden. These different assumptions and constraints provide lots of different solutions that all could be mathematically viable.

making decisions

I can use math modeling to make decisions...

## Rating and Ranking Models


Rating and ranking: Using criteria and mathematical measures as a way to rate and rank options to make decision.

For emergent math modelers, this would engage them in math discourse that might start with-
I can use math modeling to rate and rank to make decisions about ______________.
I can use math modeling to rate and rank to make decisions about the best college basketball player/team.
I can use math modeling to rate and rank to make decisions about the best vacation spot for our family.
Rating and ranking provides a great way for students to quantify the world around them. This modeling activity is primed for using collected data to make decision. Data scientists use strategies like rating and raking to make important decisions in our daily lives. Sometimes even exciting decisions like which team to rout for during the March Madnesss game 😊

# Using Math Modeling- Planning Guide

| a) What's the "Math Happening"?<br>(What do you wonder about? What do you notice?) | b) What do we already know?<br>(What do we already know that can help us?) |
| --- | --- |
| c) What do we need to know?<br>If I knew_______then, I can figure out__________. (Make assumptions.) | d) Do the math! Solve!<br>Build a Math Model or General Rule<br>(What math can I use to solve the problem?) |

e) Does the solution make sense?
(Think back to the problem statement. Does the solution work?)

F) How can you Revise, Refine and Report your solution?
(What might you change?)

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Teachers
Empowered to
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CHange in
MATHematics

**Culturally Responsive Mathematics Teaching – TM**
**Lesson Analysis Tool**

**PURPOSE:**

CRMT-TM Lesson Analysis Tool is designed to promote intentional teaching discussions and critical reflection on mathematics lessons with a combined focus on children's mathematical thinking and equity. It is not designed to be an evaluation tool of teachers but a self-reflective professional tool that can support lesson/unit design and implementation.

**TOOL DESCRIPTION:**

The **CRMT-TM Lesson Analysis Tool** consists of six important categories of mathematics teaching. Each category connects to a rubric rating scale 1-5 that provides descriptors of classroom practice including task design, implementation, and interaction. In addition, there are corresponding reflection prompts to help with lesson analysis. The table below provides a brief description of each category and accompanying reflection prompt.

| | **Category** | **Reflection Prompts** |
|---|---|---|
| **1** | **Cognitive Demand** | ***How does my lesson enable students to closely explore and analyze math concepts(s), procedure(s), and reasoning strategies?*** |
| **2** | **Depth of Knowledge & Student Understanding** | ***How does my lesson make student thinking/understanding visible and deep?*** |
| **3** | **Mathematical Discourse** | ***How does my lesson create opportunities to discuss mathematics in meaningful and rigorous ways (e.g. debate math ideas/solution strategies, use math terminology, develop explanations, communicate reasoning, and/or make generalizations)?*** |
| **4** | **Power and Participation** | ***How does my lesson distribute math knowledge authority, value student math contributions, and address status differences among students?*** |
| **5** | **Academic Language Support for ELL** | ***How does my lesson provide academic language support for English Language Learners?*** |
| **6** | **Cultural/Community-based funds of knowledge** | ***How does my lesson help students connect mathematics with relevant/authentic situations in their lives?***<br><br>***How does my lesson support students' use of mathematics to understand, critique, and change an important equity or social justice issue in their lives?*** |

**HOW TO USE:**

The best use of this tool is to promote critical discussion and reflection on math lessons with an integrated focus. It is not necessary for every single lesson to have every single category. However, the CRMT-TM lesson analysis tool does make explicit the categories of practice that should be consistently evident over time. In addition, our work with the tool suggests that categories 4-6 are less likely to be selected for lesson analysis than categories 1-3. Therefore we recommend that users of this tool be intentional in making sure that categories focusing on power and participation, academic language, and cultural funds of knowledge be examined.

To help teachers get started we suggest three strategies:

1) **Analyze a videotaped lesson using the tool**. Some good videos are publically available at www.learner.org. In pairs, rate the lesson based on evidence from the video. Discuss ratings and evidence with a colleague.
2) **Analyze a lesson plan using the tool**. Check how your lesson plan reflects these various dimensions. After your analysis, brainstorm with a colleague/coach what adaptions you can make to make the lesson more culturally responsive.
3) **Have a peer use the tool to give feedback on an observed lesson**. Select one category from categories 1-3 and one from categories 4-6. Make a conscious effort to focus your instruction and feedback based on those selected categories.

**RELATED REFERENCES:**


Aguirre, J.M. (2012) Developing Culturally Responsive Mathematics Teaching. *Fall 2012 TODOS Newsletter* TODOS- Mathematics For All. http://www.todos-math.org

Aguirre, J. M., Turner, E., Bartell, T. G., Drake, C., Foote, M. Q., & Roth McDuffie, A. (2012). Analyzing effective mathematics lessons for English learners: A multiple mathematical lens approach. In S. Celedón-Pattichis & N. Ramirez (Eds.), *Beyond good teaching: Advancing mathematics education for ELLs*. (pp. 207-222). Reston, VA: National Council of Teachers of Mathematics.

Aguirre, J. & Zavala, M. (in press). Making culturally responsive mathematics teaching explicit: A lesson analysis tool. To Appear in *Pedagogies: an International Journal*.

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Turner, E. E., Drake, C., Roth McDuffie, A., Aguirre, J. M., Bartell, T. G., & Foote, M. Q. (2012). Promoting equity in mathematics teacher preparation: A framework for advancing teacher learning of children's multiple mathematics knowledge bases. *Journal of Mathematics Teacher Education, 15*(1), 67-82. doi: 10.1007/s10857-011-9196-6.

| Category / Rating | | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **1) Cognitive Demand** | | ***Guiding Question: How does my lesson enable students to closely explore and analyze math concepts(s), procedure(s), and reasoning strategies?*** | | | | |
| | Description of rating | Students receive, recite, or memorize facts, procedures, and definitions.<br><br>There is no evidence of conceptual understanding being required.<br><br>No opportunities for mathematical analysis or exploration | Students primarily receive, recite, or perform routine procedures without analysis or connection to underlying concepts or mathematical structure.<br><br>Some opportunities for mathematical exploration, but tasks do not require analysis to complete. | There is at least one sustained activity involving analysis of procedures, concepts, or underlying mathematical structure.<br><br>There is at least 1 sustained activity that requires mathematical exploration and analysis | At least half of the lesson includes task(s) that:<br><br>• Require close analysis of procedures, concepts or underlying mathematical structure. OR<br><br>• Tasks that require significant mathematical analysis, involves complex mathematical thinking, utilizes multiple representations OR demands explanation/justification<br><br>There is evidence of sustained mathematical analysis for at least half of the lesson. | The majority of the lesson includes task(s) that require close analysis of procedures and concepts, involves complex mathematical thinking, utilizes multiple representations AND demands explanation/justification<br><br>A large majority of the lesson sustains mathematical analysis. |
| **2) Depth of Knowledge and Student Understanding** | | ***Guiding Question: How does my lesson make student thinking/understanding visible and deep?*** | | | | |
| | Description of rating | Knowledge is very thin because concepts are treated trivially or presented as non-problematic.<br><br>Students are not involved in the coverage of information they are to remember. | Knowledge remains superficial and fragmented.<br><br>Underlying or related concepts and ideas might be mentioned or covered, but only a superficial acquaintance or trivialized understanding of these ideas is evident. | Knowledge is treated unevenly during instruction.<br><br>Deep understanding of some mathematical concepts is countered by superficial understanding of some other ideas.<br><br>At least one idea may be presented in depth and its significance grasped by some (10%-20%) students, but in general the focus is not sustained. | Knowledge is relatively deep because the students provide information, arguments, or reasoning that demonstrates the complexity of one or more ideas.<br>The teacher structures the lesson so that many students (20%-50%) do at least one of the following:<br>• sustain a focus on a significant topic for a period of time;<br>• demonstrate their understanding of the problematic nature of information and/or ideas;<br>• demonstrate understanding by arriving at a reasoned, supported conclusion;<br>• explain how they solved a relatively complex problem. | Knowledge is very deep because the teacher successfully structures the lesson so that most students (50%-90%) do at least one of the following:<br>• sustain a focus on a significant topic;<br>• demonstrate their understanding of the problematic nature of information or ideas;<br>• demonstrate complex understanding by arriving at a reasoned, supported conclusion;<br>• explain how they solved a complex problem.<br>In general, students' reasoning, explanations, and arguments demonstrate fullness and complexity of understanding. |

| Rating<br>Category | | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **3) Mathematical Discourse & Communication** | ***Guiding Question:*** **How does my lesson create opportunities to discuss mathematics in meaningful and rigorous ways (e.g. debate math ideas/solution strategies, use math terminology, develop explanations, communicate reasoning, and/or make generalizations)?** | | | | | |
| | | Virtually no features of mathematical discourse and communication occur, or what occurs is of a fill-in-the-blank nature. | Sharing and the development of collective understanding among a few students (or between a single student and the teacher) occur briefly. | There is at least one sustained episode of sharing and developing collective understanding about mathematics that involves:<br>(a) a small group of students or (b) a small group of students and the teacher.<br>OR<br>brief episodes of sharing and developing collective understandings occur sporadically throughout the lesson. | There are many sustained episodes of sharing and developing collective understandings about mathematics in which many students (20%-50%) participate. | The creation and maintenance of collective understandings permeates the entire lesson.<br><br>This could include the use of a common terminology and the careful negotiation of meanings.<br><br>Most students (50%-90%) participate. |
| **4) Power and Participation** | ***Guiding Question: How does my lesson distribute math knowledge authority, value student math contributions, and address status differences among students?*** | | | | | |
| | | The authority of math knowledge exclusively resides with the teacher.<br>Mathematical contributions in lesson are almost exclusively from the teacher.<br>Teacher has final word about correct answers/solutions.<br>Student mathematical contributions are minimal.<br>Status differences among students are evident. | The authority of mathematics knowledge primarily resides with the teacher and a few students.<br>Teacher calls on/involves a few students. Their mathematical contributions by students are valued and respected.<br>Student involvement is from a particular subgroup (gender, language, ethnicity, class, disability).<br>Status differences among students remain intact and unaddressed. | The authority of math knowledge between teacher and students is **sporadically** shared.<br>At least one instance where the teacher calls on several students so that multiple mathematical contributions are accepted and valued.<br>Teacher elicits some substantive math contributions.<br>At least 1 strategy to minimize status differences among students (and specific subgroups) is evident. | The authority of math knowledge is shared between teacher and students.<br>Multiple forms of student mathematical contributions are encouraged and valued.<br>Teacher and students elicit substantive mathematics contributions.<br>Some strategies to minimize status differences among students (and specific subgroups) throughout the lesson are evident. | The authority of math knowledge is **widely** shared between teacher and students.<br><br>All mathematical contributions are valued and respected.<br><br>Student mathematical contributions are actively elicited by teacher and among students.<br><br>Multiple strategies to minimize status among students (and specific subgroups) are explicit and widespread throughout the lesson. |

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| **5) Academic Language Support for ELLs** | ***Guiding Question: How does my lesson provide academic language support for English Language Learners?*** | | | | |
| | No evidence of a language scaffolding strategy for ELLs. Students who are not yet fully proficient in English are ignored and/or seated apart from their classmates. | Although there is no explicit use of language strategies for ELLs, students' use of L1 is tolerated. Focus on correct usage of English vocabulary. | There is at least one instance in which a language scaffolding strategy is used to develop academic language (i.e., revoicing; use of cognates; translated tasks/text; use of graphic organizers; strategic grouping with bilingual students). | Sustained use of at least a couple of language strategies, such as the use of revoicing and attention to cognates, direct modeling of vocabulary, use of realia, strategic grouping of bilingual students or encouragement of L1 usage is observed at least between teacher and one, or small group, of students. | Deliberate and continuous use of language strategies, such as gesturing, use of objects (realia), use of cognates, revoicing, graphic organizers and manipulatives are observed during whole class and /or small group instruction and discussions. The main focus is the development of mathematical discourse and meaning making, not students' production of "correct" English. |
| **6a) Funds of Knowledge/Culture/Community** | ***Guiding Question: How does my lesson help students connect mathematics with relevant/authentic situations in their lives?*** | | | | |
| | No evidence of connecting to students' cultural funds of knowledge (parental/community knowledge, student interest). Lesson incorporates culturally neutral contexts that "all students" will be interested in. | There is at least one instance in connecting math lesson to community/cultural knowledge and experience. Lesson draws on student knowledge and experience. Focus is with one student or a small group of students. | There is at least one sustained episode of sharing and developing collective understanding about mathematics that involves connecting to community/cultural knowledge.<br><br>Or, brief episodes of sharing and developing collective understandings occur sporadically throughout the lesson. | There are many sustained episodes of sharing and developing collective understandings about mathematics that involves connecting to cultural/community knowledge (e.g. student experiences are mathematized, student/parent connections with math work; math examples are embedded in local community/cultural contexts and activities – i.e. games). | The creation and maintenance of collective understandings about mathematics that involves intricate connections to community/cultural knowledge and permeates the entire lesson. This would include hook/intro, main activities, assessment, closure and homework. Students are asked to analyze the mathematics within the community context and how the mathematics helps them understand that context. |
| **6b) USE of critical knowledge/social justice Support** | ***Guiding Question: How does my lesson support students' use of mathematics to understand, critique, and change an important equity or social justice issue in their lives?*** | | | | |
| | No evidence of connection to critical knowledge (socio-political contexts, issues that concern students) | Opportunity to critically mathematize a situation went unacknowledged or unaddressed when present. | There is at least one instance of connecting mathematics to analyze a sociopolitical/cultural context. | There is at least one major activity in which students collectively engage in mathematical analysis within a sociopolitical/authentic or problem-posing context. Mathematical arguments are provided to solve the problems. Pathways to change/transform the situation are briefly addressed. | Deliberate and continuous used of mathematics as an analytical tool to understand an issue/context, formulate mathematically-based arguments to address the issues and provide substantive pathways to change/transform the issue. |

# The TRU Math Conversation Guide: A Tool for Teacher Learning and Growth[1]

Teachers' work is remarkably complex—in particular when the goal is not just to present facts and formulas for students to memorize, but to **T**each for the **R**obust **U**nderstanding of **Math**ematics. Because of this, there is *always* room for learning and growth, for *every* teacher regardless of prior training, years of experience, or current successes. Indeed, ongoing learning is the essence of teaching.

Our experience as teachers, coaches, and researchers has been that our most meaningful learning occurs when we interact with others, developing and sustaining relationships that simultaneously challenge and support us. These relationships push us to expand our vision of teaching and learning. They offer perspectives on our work that differ from our own. And they respect our intelligence, skill, and intentions—as well as our need to continually grow. These supportive relationships not only help us to alter our practice but also to deepen our understanding of the complex work we are undertaking.

Unfortunately, much of the professional development that we have experienced has focused less on these aspects of learning and more on "experts" sharing "best practices" that we are supposed to simply import to our own classrooms. Here, we have tried to create a professional development tool that builds on what teachers, coaches, and professional learning communities know.

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[1] This is a working document. We hope that reflecting on teaching in the ways suggested here will be productive. We also welcome comments and suggestions for improvement. Please contact Evra Baldinger (evra@berkeley.edu) and Nicole Louie (nllouie@utep.edu) with your feedback.

This Conversation Guide is part of a collection of tools and papers supporting Teaching for Robust Understanding, which are are available at http://map.mathshell.org/trumath.php. These tools include a domain-general version of the Conversation Guide as well as an algebra-specific version. More details on each dimension in the framework and the research base behind it are provided on the website. An updated version of this Conversation Guide may also be available online.

Work on TRU is the product of The Algebra Teaching Study (NSF Grant DRL-0909815 to PI Alan Schoenfeld, U.C. Berkeley, and NSF Grant DRL-0909851 to PI Robert Floden, Michigan State University), and of The Mathematics Assessment Project (Bill and Melinda Gates Foundation Grant OPP53342 to PIs Alan Schoenfeld, U.C. Berkeley, and Hugh Burkhardt and Malcolm Swan, The University of Nottingham).

Suggested citation: Baldinger, E. M., Louie, N., & the Algebra Teaching Study and Mathematics Assessment Project. (2014). *The TRU Math conversation guide: A tool for teacher learning and growth*. Berkeley, CA & E. Lansing, MI: Graduate School of Education, University of California, Berkeley & College of Education, Michigan State University. Retrieved from: http://TRU.berkeley.edu.

# FIVE DIMENSIONS OF POWERFUL CLASSROOMS

This Conversation Guide represents our best efforts to use research to support teacher learning and growth in a way that accounts for both how people learn and the complexity of teaching practice. Instead of prescribing instructional techniques or tricks, we offer a set of questions organized around five dimensions of teaching identified by research as critical for students' mathematics learning.

The dimensions are summarized in the table below. Together, they offer a way to organize some of the complexity of teaching so that we can focus our learning together in deliberate and useful ways. They include attention to content, practices, and students' developing identities as thinkers and learners. There is necessarily some overlap between dimensions; rather than capturing completely distinct categories, each dimension is like a visual filter, highlighting different aspects of the same phenomena in everyday classroom life. We encourage you to think about interactions between dimensions when it is useful for you. The questions on subsequent pages of the Guide will also direct your attention to particular kinds of overlap.

| The Five Dimensions of Mathematically Powerful Classrooms | |
|---|---|
| **The Mathematics** | How do mathematical ideas from this unit/course develop in this lesson/lesson sequence? How can we create more meaningful connections? |
| **Cognitive Demand** | What opportunities do students have to make their own sense of mathematical ideas? To work through authentic challenges? How can we create more opportunities? |
| **Equitable Access to Content** | Who does and does not participate in the mathematical work of the class, and how? How can we create more opportunities for each student to participate meaningfully? |
| **Agency, Ownership, and Identity** | What opportunities do students have to see themselves and each other as powerful mathematical thinkers? How can we create more of these opportunities? |
| **Formative Assessment** | What do we know about each student's current mathematical thinking? How can we build on it? |

# WHAT THE CONVERSATION GUIDE IS FOR

The purpose of this Conversation Guide is not to tell anyone how to teach, but to facilitate *coherent* and *ongoing* discussions in which teachers, administrators, coaches, and others *learn together*. We hope that the questions in the Conversation Guide will support educators with different experiences, different expertise, and different strengths to work together to develop a common vision, common priorities, and common language, in order to collaboratively improve instruction and better support students to develop robust understandings.

The Conversation Guide can be used to support many different kinds of conversations, including (but not limited to):

- Conversations to develop common vision and priorities across groups of teachers (within the same school and/or across different schools)
- Conversations between teachers and administrators and instructional coaches around classroom observations (see also the TRU Observation Guide, available at http://map.mathshell.org/trumath.php)
- Conversations between teachers around peer observations
- Conversations around video recordings of mathematics teaching and learning
- Conversations about planning a particular unit or lesson
- Conversations about a particular instructional strategy or set of strategies
- Ongoing individual reflection

We have found that the Conversation Guide can be useful for facilitating a one-time conversation. *But its real power lies in its support for creating coherence across conversations.* The Guide can help individuals as well as groups of educators to set an agenda and work on it consistently over time. For example, a teacher team (such as a math department) might decide to spend a semester focusing on issues of Equitable Access to Content (Dimension 3). Meeting time might then be spent reflecting on the kinds of access that are currently available to students and planning lessons with the goal of monitoring and expanding access in mind, using the Equitable Access to Content questions and prompts in this Guide. Members of the team might observe each other's classrooms with a focus on these same questions and prompts. The principal might find ways to support teachers to attend workshops related to the theme of Equitable Access to Content, rather than supporting a series of disconnected trainings.

In the remainder of this document we provide an overview of each dimension; discussion questions for each dimension, for your use in reflecting on and planning instruction; and a set of suggestions for how to use the discussion questions.

We hope you will find the Conversation Guide useful. Happy teaching and learning!

# HOW TO USE THIS CONVERSATION GUIDE

Our field tests and our experiences as instructional coaches have led to a few suggestions that may help you make the most of this conversation guide. In this section, we share these suggestions and give some examples of how conversations using the guide might look.

## 1. Set a long-term learning agenda.

Complex learning—like learning how to teach for robust student understanding—has so many facets that it is easy to jump from one thing to another, without making clear progress on anything. Setting a long-term learning agenda can help us focus our energies, whether we're full-time classroom teachers or people who support classroom teachers. Opportunities to have deep conversations about practice are few and far between, but if we have a core learning agenda that we can return to again and again, we stand a better chance of leveraging all our strengths to learn together about something that matters.

The process of setting an agenda can unfold in many different ways. Various stakeholders may come in with clear (and perhaps competing) ideas about what they want to focus on, or it may happen that no one has a particular preference. Whatever the case may be, it is important that all participants, especially classroom teachers, feel connected to the learning agenda. Our learning is much more powerful when we get to learn about things that trouble or inspire us.

Some examples of long-term learning agendas might be, "This semester, I want to focus on getting students to share their reasoning, not just answers or steps," or "This year, I want to get better at engaging students who get frustrated and give up easily." As you set your own learning agenda, it may be useful to read through the dimensions and discussion questions, to see if anything jumps out as particularly important or exciting.

## 2. Use the discussion questions like a menu. Pick and choose.

You might have noticed that there are a lot of questions in this Guide! Our design assumes that you WILL NOT try to discuss every bullet, one by one, each time you use the guide. Instead, we hope you will identify areas of the guide that are appropriate for your learning agenda and return to these areas again and again. We expect that some of the questions will be difficult to answer—and that by discussing them together you will find new ways of understanding teaching and learning and come up with ideas for things to try in order to improve both.

## 3. Ground discussion in specific, detailed evidence.

We've all made statements like, "My kids seem to really get linear equations" or "They're really struggling with fractions." While these statements convey a picture of student understanding in a quick and concise way, they need to be followed up with more detailed information. Otherwise, it is difficult to make instruction responsive to student thinking, and easy to miss opportunities to build on students' strengths or address their misconceptions. One way to make our observations more specific is to talk about content with as much detail as possible; for example, instead of saying “My kids are really struggling with fractions,” you might observe that “Even though I’ve seen my kids do just fine with finding equivalent fractions and even adding them, they just seem to shut down every time they see a fraction,” or “they’re reducing fractions in a mechanical way, but they don’t seem to *see* that 4/6 of a chocolate bar and 2/3 of a chocolate bar represent the same amount.”

Pressing for specific examples also makes observations more accurate and concrete, helping us get away from our general impressions and closer to actual student thinking. Talking about specific students—and ways that their thinking is or isn’t typical of the class—is another strategy. Not only does this strategy give us a more detailed and accurate picture of the thinking that is going on in our classrooms, but it also opens up instructional possibilities. For example, noticing that today, Jessica drew a really helpful picture to represent fractions could lead you to invite Jessica to share her method with the rest of class, creating a learning opportunity that is invisible in “They’re really struggling with fractions.”

Finally, attending to particular students can help us think about patterns of marginalization in society at large (e.g., fewer resources for ELLs, or stereotypes that link race, gender, and mathematics ability), and how our classrooms might work to replicate or counter those patterns for our own students.

If you are able to ground your conversations in shared experiences of the same classroom (from peer observations, co-teaching, instructional coaching, etc.), you will benefit from more eyes and more perspectives on the details of classroom activity. But even if this isn’t possible in any particular conversation, working with evidence of specific students’ thinking and understanding will make your conversation a richer resource for your own learning.

## 4. If you are conducting a classroom observation, pre-brief.

Classroom observations are generally accompanied by a debrief conversation. Pre-brief conversations can be just as important. If you can, have a conversation prior to each observation. In this conversation, clarify the goals not just for the lesson, but also for the observation of the lesson. Talk about goals for students, so that observers can understand what the teacher is trying to accomplish. Also remind each other of the teacher’s learning agenda so that you can discuss how the observer can be most helpful. We have found this question especially useful: “What do we want to be able to talk about in our debrief conversation?” From there, you might discuss what the observer should be looking for (e.g., recording the questions the teacher asks, or focusing on a particular student), and what kinds of interactions (if any) the observer should have with students.

The pre-brief conversation is one way of capitalizing on the focus and organization that a learning agenda offers. Without it, it’s easy to get distracted during the observation. It’s also easy for the observer to notice things that are not interesting or important to the teacher, which are less likely to help the teacher learn and grow.

The Conversation Guide includes prompts for planning, which can be used for planning observations as well as for planning lessons. Discussing these prompts should bring to the surface ideas about what is likely to happen in the lesson, given the tasks students will be given, the participation structures that will be used, and so on. This kind of anticipatory thinking might lead to tweaks in the lesson plan, but just as important, it can establish common focus between the teacher and observer. This adds richness to the debrief after the lesson; everyone can then reflect on the ways that things worked out the way they were intended to, ways they were surprising, and next steps in light of that information.

## 5. Link planning to reflection and vice versa.

This Guide includes prompts for "planning" and prompts for "reflecting." We do not mean to suggest that you restrict each conversation to a focus on one or the other; rather, it will be useful to connect these perspectives in many conversations. Reflection is most practical when it leads to next steps, and next steps (planning) should be firmly grounded in reflection on what has already happened. It is worthwhile to make space for thinking about what has already happened without jumping to next steps too quickly, however. Reflecting on the details of what we have observed opens up possibilities for future action that might otherwise remain hidden (as described above). In addition, different people see different things, and sharing our observations can enrich everyone's understanding of what students have been doing, thinking, and learning.

## 6. Work from teachers' strengths.

Our culture often prompts us to focus on our weaknesses, and on the areas where we need improvement. But our *strengths* are huge assets when it comes to learning and improving our practice. Knowing our strengths supports us to engage with challenges, giving us a starting point to work from and a reason to believe that we can be successful. Identifying teachers' strengths, making them explicit, and using them as authentic resources for growth can therefore support teachers to think deeply and critically about their practice, to strive for improvement, to actually improve by building on their strengths, and to develop productive relationships with supportive others, all at once.

In practice, this might mean prompting teachers (not just supervisors) to share *their* observations, interpretations, and ideas for moving forward; creating diverse opportunities to identify what teachers already do well, including planning together, reflecting together, and observing various kinds of interactions with students (e.g., leading discussions, intervening at small groups, and building rapport with individual students); and building next steps around strengths instead of deficits (e.g., working on supporting students who have been reluctant to participate by building on a teacher's skill at noticing something that each student is good at).

# The Mathematics

*Core Questions: How do mathematical ideas from this unit/course develop in this lesson/lesson sequence? How can we create more meaningful connections?*

Students often experience mathematics as a set of isolated facts, procedures and concepts, to be rehearsed, memorized, and applied. Our goal is to instead give students opportunities to experience mathematics as a coherent and meaningful discipline. This requires identifying the important mathematical ideas behind facts and procedures, highlighting connections between skills and concepts, and relating concepts to each other—not just in a single lesson, but also across lessons and units. It requires engaging students with centrally important mathematics in an active way, so that they can make sense of concepts and ideas for themselves and develop robust networks of understanding. And it requires engaging students in authentic performances of important disciplinary practices (e.g., reasoning abstractly and quantitatively, constructing mathematical arguments and critiquing the reasoning of others).

## Planning

How will important mathematical ideas and practices develop in this lesson and unit? How can we connect the ideas and practices that have surfaced in recent lessons to this lesson and future lessons?

## Reflecting

How have we seen students engage with important mathematical ideas and practices? How has this engagement looked and sounded in specific cases?

## Things to think about

- What are the mathematical goals for the lesson?
- What connections exist (or could exist) between important ideas in this lesson and important ideas in past and future lessons?
- How do important mathematical practices develop in this lesson/unit?
- How are facts and procedures in the lesson justified?
- How are facts and procedures in the lesson connected with important ideas and practices?
- How do we see/hear students engage with important ideas and practices during class?
- Which students get to engage deeply with important ideas and practices?
- How can we create opportunities for more students to engage more deeply with important ideas and practices?

# Cognitive Demand

*Core Questions: What opportunities do students have to make their own sense of mathematical ideas? How can we create more opportunities?*

We want students to engage authentically with important mathematical ideas, not simply receive knowledge. This kind of learning requires that students engage in *productive struggle,* grappling with difficult concepts and challenging problems. As teachers, we must support students in ways that maintain their opportunities to do this grappling for themselves. Our goal is to help students understand the challenges they confront and persist in solving them, while leaving them room to make their own sense of those challenges.

## Planning

What opportunities might students have to make their own sense of important mathematical ideas? How can we create more of these opportunities?

## Reflecting

How have we seen students make their own sense of important mathematical ideas? How has this sense-making looked and sounded in specific cases?

## Things to think about

- What opportunities exist for students to struggle with important mathematical ideas?
- How are students' struggles supporting their engagement with important mathematical ideas?
- How does (or how could) the teacher respond to students' struggles, and how do (or how could) these responses maintain students' opportunities to develop their own ideas and understandings?
- What resources (other students, the teacher, notes, texts, technology, manipulatives, various representations, etc.) are available for students to use when they encounter struggles? Are there more resources we can make available?
- What resources are students actually using, and how might they be supported to make better use of resources?
- Which students get to engage deeply with important mathematical ideas?
- How can we create opportunities for more students to engage more deeply with important mathematical ideas?
- What community norms seem to be evolving around the value of struggle and mistakes?

# Equitable Access to Content

*Core Questions: Who does and does not participate in the mathematical work of the class, and how? How can we create more opportunities for each student to participate meaningfully?*

All students should have access to opportunities to develop their own understandings of rich mathematics, and to build productive mathematical identities. For any number of reasons, it can be extremely difficult to provide this access to everyone, but that doesn't make it any less important! We want to challenge ourselves to recognize who has access and when. There may be mathematically rich discussions or other mathematically productive activities in the classroom—but who gets to participate in them? Who might benefit from different ways of organizing classroom activity?

## Planning

What opportunities exist for each student to participate in the intellectual work of the class? How can we create more opportunities for more students?

## Reflecting

Who have we seen participate in the intellectual work of the class? How has this participation looked and sounded in specific cases?

## Things to think about

- What is the range of ways that students can and do participate in the mathematical work of the class (talking, writing, leaning in, listening hard; manipulating symbols, making diagrams, interpreting text, using manipulatives, connecting different ideas, etc.)?
- Which students participate in which ways?
- Which students are most active, and when?
- In what ways can particular students' strengths or preferences be used to engage them in the mathematical activity of the class?
- What opportunities do various students have to make meaningful mathematical contributions?
- What are the language demands of participating in the mathematical work of this class (e.g., academic vocabulary, mathematical discourse practices)?
- How can we support the development of students' academic language?
- How are norms (or interactions, lesson structures, task structure, particular resources, etc.) facilitating or inhibiting participation for particular students?
- What teacher moves might expand students' access to meaningful participation (such as modeling ways to participate, holding students accountable, point out students' successful participation)?
- How can we support particular students we are concerned about (in relation to learning, issues of safety, participation, etc.)?
- How can we create opportunities for more students to participate more actively?

# Agency, Ownership, and Identity

*Core Questions: What opportunities do students have to see themselves and each other as powerful doers of mathematics? How can we create more of these opportunities?*

Many students have negative beliefs about themselves and mathematics, for example, that they are "bad at math," or that math is just a bunch of facts and formulas that they're supposed to memorize. Our goal is to support all students—especially those who have not been successful with mathematics in the past—to develop a sense of mathematical agency and ownership over their own learning. We want students to come to see themselves as mathematically capable and competent—not by giving them easy successes, but by engaging them as sense-makers, problem solvers, and creators of mathematical ideas.

## Planning

What opportunities might exist for students to generate and explain their own ideas? To respond to each other's ideas? How can we create more opportunities?

## Reflecting

How have we seen students explain their own and respond to each other's ideas? What has that looked and sounded like in specific cases?

## Things to think about

- Who generates the ideas that get discussed?
- What kinds of ideas do students have opportunities to generate and share (strategies, connections, partial understandings, prior knowledge, representations)?
- Who evaluates and/or responds to others' ideas?
- How deeply do students get to explain their ideas?
- How does (or how could) the teacher respond to student ideas (evaluating, questioning, probing, soliciting responses from other students, etc.)?
- How are norms about students' and teachers' roles in generating ideas developing?
- How are norms about what counts as mathematical activity (justifying, experimenting, connecting, practicing, memorizing, etc.) developing?
- Which students get to explain their own ideas? To respond to others' ideas in meaningful ways?
- Which students seem to see themselves as powerful mathematical thinkers right now?
- How might we create more opportunities for more students to see themselves and each other as powerful mathematical thinkers?

# Formative Assessment

*Core Questions: What do we know about each student's current mathematical thinking? How can we build on it?*

We want instruction to be responsive to students' actual thinking, not just our hopes or assumptions about what they do and don't understand. It isn't always easy to know what students are thinking, much less to use this information to shape classroom activities—but we can craft tasks and ask purposeful questions that give us insights into the strategies students are using, the depth of their conceptual understanding, and so on. Our goal is to then use those insights to guide our instruction, not just to fix mistakes but to integrate students' understandings, partial though they may be, and build on them.

## Planning

What do we know about each student's current thinking, and how might this lesson or unit build on that thinking? How can we learn more about each student's thinking?

## Reflecting

What have we learned in recent lessons about each student's thinking? How did this thinking look and sound in specific cases? How was this thinking built upon?

## Things to think about

- What opportunities exist (or could exist) for students to develop their own strategies, approaches and understandings of mathematics?
- What opportunities exist (or could exist) for students to share their ideas and reasoning and to connect their ideas to others'?
- What different ways do students get to share their mathematical ideas and reasoning (writing on paper, speaking, writing on the board, creating diagrams, demonstrating with materials/artifacts, etc.)?
- Who do students get to share their ideas with (a partner, a small group, the whole class, the teacher)?
- What opportunities exist to build on students' mathematical thinking, and how are teachers and/or other students taking up these opportunities?
- How do students seem to be making sense of the mathematics in the lesson, and what responses might build on that thinking?
- How can activities be structured so that students have more opportunity to build on each other's ideas?
- What might we try (what tasks, lesson structures, questioning prompts, etc.) to surface student thinking, especially the thinking of students whose ideas we don't know much about yet?

# A NOTE: What We Mean By "Important Mathematical Ideas and Practices"

"Important mathematical ideas" are notoriously hard to define. Which ideas are important? Which are not? What even counts as an "idea"? Who should have the authority to decide? Our intention with the Conversation Guide is to support discussions about these questions rather than to offer answers. To us, it is much more important to work together to push our students and ourselves as educators toward more interconnected and fundamental understandings of mathematics than to decide exactly which ideas are most important. This pushing is crucial, because traditional views of school mathematics—and many of today's textbooks and standards documents—define mathematics in terms of isolated topics, skills, and sub-skills. Thinking about the progression of mathematical ideas as "Day 1: Add and Subtract Fractions With Like Denominators; Day 2: Multiply Fractions; Day 3: Divide Fractions; Day 4: Add and Subtract Fractions with Unlike Denominators" (a typical textbook progression) makes it difficult to develop conceptual understanding and a sense of meaning behind all of the mechanics. This is both untrue to mathematics as a discipline and alienating for many students.

One way of finding connections among apparently isolated topics is to focus on core mathematical practices. For example, *constructing an argument* is one such practice. Creating opportunities for students to develop skill in constructing mathematical arguments can bridge the otherwise disparate topics that math courses are typically supposed to cover. (Note the differences between a skill like constructing an argument and a skill like adding fractions.) Yet a focus on core practices does not eliminate the need to identify important mathematical ideas and use these ideas to organize instruction.

We find the questions below useful for shifting our focus from facts and procedures to important mathematical ideas. We hope they will be helpful for you as well.

- What do we want students to understand about the relevant mathematical objects (fractions, negative numbers, the coordinate plane, triangles, etc.) in this lesson? In this unit?
- What mathematical relationships, patterns, or principles do we want students to understand in this lesson? In this unit?
- How might students connect math ideas in this lesson/unit with ideas that came before or will come later? Are there overarching principles or relationships or patterns that they might work toward understanding?
- What are different ways of representing the math in this lesson/unit? How might different representations be connected to each other and how might these connections deepen our students' understanding?
- How do the ideas we're considering develop across multiple lessons/units?
- What are some ways to make connections to this idea in different lessons/units/content areas?

Some examples of math ideas that might be considered "important":

- Area and perimeter are fundamentally different measurable attributes of two-dimensional shapes. It is possible to change shapes such that neither, one, or both of these attributes change. For some families of shapes, there are interesting relationships between them.
- Relationships between two variables can be represented using equations, tables, graphs, and verbal descriptions. Parameters of the relationship between the variables (e.g., the rate of change) can be identified in each of these representations and connected across representations.

- Right triangles have special properties that are different from the properties of other triangles. These properties give us special access to information about things like angle measures and side lengths in particular right triangles.
- Many sets of changing quantities are proportionally related. This means that certain aspects of the relationship are constant and unchanging, which allows us to use the relationship to determine one quantity given the other.

One characteristic of all of these ideas is that they go beyond naming topics and skills. For example, we might know that we want to "cover proportional relationships" in a particular unit, or that we want students to be able to solve proportions. However, without consideration of the important underlying ideas that we want our students to make sense of, we are likely to get lost in facts and procedures. We are likely to miss opportunities to support students to build conceptual understandings, to make connections, and to develop a sense of themselves as powerful learners and thinkers.

Our hope is that as teachers and others think together about teaching, they can continuously push each other to think about the mathematics that students need to learn in bigger, deeper, richer, and more interconnected ways. So while our discussion questions frequently refer to "important mathematical ideas" as though there were a set list of such ideas somewhere that you could simply consult, we hope that you will instead find ways to explore and interrogate what "important mathematical ideas" means to you.